A POLYNOMIAL FORMULA FOR FIBONACCI NUMBERS

These examples demonstrate a technique for obtaining a polynomial that generates any finite sequence of Fibonacci numbers. The leading order differences must be calculated in order to determine the polynomial, but they follow a discernible pattern. The resulting polynomial generates only those terms in the initial sequence and is useful in some applications.

Acknowledgment

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References

- 1. S. J. Bezuszka, L. D'Angelo, & M. J. Kenney. Applications of Finite Differences. Chestnut Hill, MA: Boston College Press, 1982.
- 2. Brother Alfred Brousseau. "Formula Development through Finite Differences." Fibonacci Quarterly 16.1 (1978):53-67.
- 3. Allen W. Smith. *Elementary Numerical Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1986.

(continued from page 140)

The reviewer has some problems with comments made by the authors. First, the authors could, I believe, have misinterpreted the quote by Schalau and Opolka which is given in the Foreword. The Pythagorean triple problem was completely solved in antiquity if by this statement Schalau and Opolka meant that a method had been developed which totally solved the problem of finding all Pythagorean triples. If Schalau and Opolka meant that no new results could be found, then the authors are correct. I believe that the former is the case.

The authors also claim that there is no technique for systematically generating all Pythagorean triples by the old method. This is, I believe, a matter of opinion. The reviewer happens to believe that the original technique developed by Diophantus is very systematic. That is, (x, y, z) is a Pythagorean triple if and only if $x = u^2 - v^2$, y = 2uv, and $z = u^2 + v^2$, where u > v. The problem here is the meaning of "systematic." The authors also feel that their method is more time efficient. The reviewer has a problem with this. Finding the greatest common divisor of two integers, even when large, is not a problem for the computer. It does take time but would it take any more time than is needed to go through the contraction method developed by the authors or to find the convergents needed for the continued fraction or to pick and implement the method (class) that gives the correct value of n? I think not.

Overall, I would recommend the book and suggest that those interested in Pythagorean triples or Pellian equations read it.

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