recurrence (4) would give two different values of $u_{k,n}$, both of which cannot be equal to $u_{i,n}$. Consequently, there are at most

$$5(k-1)\left(\frac{\log 4S_k}{\log \alpha}\right)^2$$

values of $u_{k,2} \in (M_k, S_k] \cap \mathbf{N}$ such that $u_{k,n} = u_{i,m}$ for some $n, m, 1 \le i \le k-1$. Therefore, the number of values $u_{k,2} \in (M_k, S_k] \cap \mathbf{N}$ such that $u_{k,n} \ne u_{i,m}$ for all $n, m, 1 \le i \le k-1$ is at least

$$S_k - M_k - 5(k-1)\left(\frac{\log 4S_k}{\log \alpha}\right)^2,$$

which is positive by (11), and hence the choice of such an $u_{k,2}$ is possible.

This induction on k shows that there are infinitely many sequences $(u_{k,n})_{n\in\mathbf{N}}$. Every natural number occurs in one of these sequences by (7). It occurs exactly once by property (P) which holds for these sequences.

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GENERALIZED PASCAL TRIANGLES AND PYRAMIDS
THEIR FRACTALS, GRAPHS, AND APPLICATIONS

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