

# THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

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## **PURPOSE**

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for wide-spread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## **EDITORIAL POLICY**

THE FIBONACCI QUARTERLY seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

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Articles should be submitted using the format of articles in any current issues of **THE FIBONACCI QUARTERLY.** They should be typewritten or reproduced typewritten copies, that are clearly readable, double spaced with wide margins and on only one side of the paper. The full name and address of the author must appear at the beginning of the paper directly under the title. Illustrations should be carefully drawn in India ink on separate sheets of bond paper or vellum, approximately twice the size they are to appear in print. Since the Fibonacci Association has adopted  $F_1 = F_2 = 1$ ,  $F_n + I = F_n + F_n - I$ ,  $n \ge 2$  and  $L_1 = 1$ ,  $L_2 = 3$ ,  $L_n + I = L_n + L_n - I$ ,  $n \ge 2$  as the standard definitions for The Fibonacci and Lucas sequences, these definitions should not be a part of future papers. However, the notations must be used. One to three complete A.M.S. classification numbers must be given directly after references or on the bottom of the last page. Papers not satisfying all of these criteria will be returned. See the worldwide web page at:

http://www.sdstate.edu/~wcsc/http/fibhome.html.

Three copies of the manuscript should be submitted to: CURTIS COOPER, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, CENTRAL MISSOURI STATE UNIVERSITY, WARRENSBURG, MO 64093-5045.

Authors are encouraged to keep a copy of their manuscripts for their own files as protection against loss. The editor will give immediate acknowledgment of all manuscripts received.

The journal will now accept articles via electronic services. However, electronic manuscripts must be submitted using the typesetting mathematical wordprocessor AMS-TeX. Submitting manuscripts using AMS-TeX will speed up the refereeing process. AMS-TeX can be downloaded from the internet via the homepage of the American Mathematical Society.

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# The Fibonacci Quarterly

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