FIBONACCI STATISTICS IN CONIFERS BROTHER ALFRED BROUSSEAU St. Mary's College, California

The Editor of the Fibonacci Quarterly has received an urgent phone call from a Houghton-Mifflin representative: "Is the picture of the pine cone in your manuscript spiralling correctly?" The thought was that possibly the negative had been turned over and so what should be steep spirals going to the left would become steep spirals going to the right. The Editor relayed the question to the Managing Editor who hurried to the basement, picked up a pine cone and found on the first try that the direction of the spirals agreed with the picture.

Another life situation. After giving a talk on Fibonacci numbers in nature or exhibiting specimens which show the spirals and Fibonacci numbers, the query naturally arises: "How constant are these numbers in nature?"

With such questions in mind, an investigation was begun in the summer of 1969. Very quickly it was discovered that spirals on cones go in both directions. For example, if we consider two particular sets of spirals, one steep and the other more gradual where the count from one intersection to the next along the spirals is eight on one spiral and five on the other, then on some cones the steep spiral goes to the right and the more gradual spiral goes to the left, while on others, it is just the reverse.

This led to the following general approach. Wherever possible cones would be studied for individual trees; approximately four hundred cones would be examined for each species. The information and results for the various species are set forth in the remainder of this article.

LODGEPOLE PINE (Pinus Murrayana), also known as Tamarack Pine

The cones on this tree are small and abundant. They were collected in the neighborhood of Huntington Lake in the middle Sierra. Because they are open and difficult to count in this state they were soaked in water to close them after which it was relatively easy to follow the spirals.

In this report and those that follow the notation SR means that the count along the gradual spiral from one intersection of the two spirals to the next was 8, while that along the steep spiral was 5. Thus in the 8R case the gradual

spiral goes to the right. It should be noted that this method of counting simply reflects the fact that there are eight steep spirals and five gradual spirals on the cone (i. e., of the spirals we are considering). NS means non-standard: it was not possible to find the 8-5 pattern on cones listed under this heading. This does not mean that in all cases there was no Fibonacci pattern: sometimes there was double a Fibonacci number, for example. But we are not interested in these deviants as such, but simply in their relative abundance.

TREE	8R	8L	\mathbf{NS}	%8 m R	%8L	% NS
1	95	68	4	56.9	40.7	2.4
2	84	70	1	54.2	45.2	0.6
Various	285	282	3			
TOTAL	464	420	8	52.0	47.1	0.9

JEFFREY PINE (Pinus Jeffreyi)

These large cones were collected in the vicinity of Huntington Lake. The count was made after they were closed by soaking.

TREE	8R	8L	\mathbf{NS}	%8R	%8L	$\% \mathrm{NS}$
1	42	40	1	50.6	48.2	1.2
2	21	22	1	47.7	50.0	2.3
3	38	38	13	42.7	42.7	14.6
Various	90	93	3			
TOTAL	191	193	18	47.5	48.0	4.5

SUGAR PINE (Pinus Lambertiana)

The cones were studied on the spot in the area west of Kaiser Peak in the middle Sierra region. In many cases, due to the fact that they were not closed it was not possible to determine whether they had the pattern or not. Thus these cones do not provide positive information on the presence or absence of the given pattern.

TREE	8R	8L	\mathbf{NS}	%8R	%8L	$\% \mathrm{NS}$
1	25	28		47.2	52.8	
2	60	29	1	66.7	32.2	1.1
3	5 7	53		51.8	48.2	
Various	68	80	1			
TOTAL	210	190	2	52.2	47.3	0.5

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It should be noted that this is a very regular cone and that only in the few cases noted was there positive evidence of the lack of the usual pattern.

SILVER PINE (Pinus monticola)

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The cones were collected on Kaiser Ridge not far from Huntington Lake. They were soaked so as to make it possible to follow the spirals conveniently. The count was five along the gradual spiral and three along the steep spiral from one intersection to the next.

8R	8L	\mathbf{NS}	$\%8\mathrm{R}$	%8L	$\% \mathrm{NS}$
15	16		48.4	51.6	
26	33		44.1	55.9	
48	56		46.2	53.8	
64	65	5	47.8	48.5	3.7
65	56				
218	226	5	48.6	50.3	1.1
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YELLOW PINE (Pinus ponderosa)

The cones were collected in the middle Sierra between Auberry and Pine Ridge. They were soaked before the cones were examined.

TREE	$8\mathbf{R}$	8L	\mathbf{NS}	%8 m R	%8L	$\% \mathrm{NS}$
1	44	52		45.8	54.2	S1
2	59	46	2	55.1	43.0	1.9
3	74	37		66.7	33.3	
4	35	58		37.6	62.4	
Various	3	19				
TOTAL	215	212	2	50.1	49.4	0.5

ONE-NEEDLED PINYON (Pinus monophylla)

There is a fine stand of these trees about eight miles from Tioga Pass on the east side of the Sierra. In one notable case it was possible to study 140 fresh cones on the tree. Cones picked up from the ground were soaked before the count was made. The best way to study this cone is when it is fresh and green. Often after closing with water the old cones tend to retain some of their irregularities.

(See table on next page.)

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	TREE	5R	5L	NS	$\%5\mathrm{R}$	$\%5{ m L}$	$\%\mathrm{NS}$
	1	6	8		42.9	57.1	
	2	2	5		28.6	71.4	
	3	67	72	1	47.9	51.4	0.7
	4	18	18		50.0	50.0	
	Various	107	97	4			
	TOTAL	200	200	5	49.4	49.4	1.2

FOXTAIL PINE (Pinus Balfouriana)

Some forty miles south of Bishop is the town of Independence. Thirteen miles west of this township at over 9,000 ft. is a spot known as Onion Valley. It was there that specimens of foxtail pine cones were collected. They were soaked before the count was made and hence this species provides evidence of exceptions to the regular pattern.

	5R	5L	\mathbf{NS}	$\%5\mathrm{R}$	%5L	%NS
TOTAL	212	212	36	46.1	46.1	7.8

LIMBER PINE (Pinus flexilis)

These cones were examined on the spot at Onion Valley. Cones in which the pattern could not be discerned were simply not considered and hence these statistics give no evidence regarding exceptions.

	5R	5L	$\%5\mathrm{R}$	%5L
TOTAL	226	182	55.4	44.6

BRISTLECONE PINE (Pinus aristata)

About fifteen miles below Bishop just above Big Pine there is a turnoff leading to the Ancient Bristlecone Pine Area. Since this is a protected area under the control of the Forest Service, it was necessary to study the cones on the spot. Those on which the pattern could not be discerned were eliminated from consideration and hence the following statistics give no evidence regarding possible exceptions.

(See table on next page.)

1969]	FIBONACCI STATISTICS IN CONIFERS					
TREE	8R	8L	%8R	% 8L		
1	27	23	54.0	46.0		
2	3	11	21.4	78.6		
3.	24	18	57.1	42.9		
4	5	5	50.0	50.0		
5	13	23	36.1	63.9		
6	9	7	56.3	43.7		
Various	93	88				
TOTAL	174	175	49.9	50.0		

DIGGER PINE (Pinus Sabiniana)

About 70 of these cones were found near Auberry, 30 on Mt. Diablo, and approximately 225 on Mt. Hamilton. These are very large cones and it would have been quite difficult to collect them, soak them, and thus arrive at positive evidence regarding exceptions. Hence they were counted on the spot, the uncountable specimens not being given consideration.

TREE	$8\mathbf{R}$	8L	%8R	%8L
1	6	10	37.5	62.5
2	7	15	31.8	68.2
3	3	11	21.4	78.6
4	12	19	38.7	61.3
Various	135	121		
TOTAL	163	176	48.1	51.5

COUNTER PINE (Pinus Coulteri)

About fifty of these cones were examined on Mt. Diablo and the rest on Mt. Hamilton and its vicinity. Again, these cones are very large and it would have been quite a problem to collect them, soak them, and thus get positive evidence regarding deviations from the usual pattern.

TREE	8R	8L	$\%8\mathrm{R}$	%8L
1	9	6	60.0	40.0
Various	159	152		
TOTAL	168	158	51.5	48.5

KNOBCONE PINE (Pinus attenuata)

About 140 of these cones were examined at St. Mary's College (Contra Costa County) and the rest were found west of Redding. Most of the cones were countable but those that were open were collected and soaked before the count was made. Thus the fact that there are no exceptions to the pattern is significant in this case.

TREE	8R	8L	%8R	%8L
1	3	19	13.6	86.4
2	4	5	44.4	55.6
3	22	10	68.8	31.2
4	14	8	63.6	36.4
5	6	47	11.3	88.7
6	9	1	90.0	10.0
7	13	8	61.9	38.1
8	8	4	66.7	33.3
9	4	6	40.0	60.0
10	7	10	41.2	58.8
11	10	33	23.3	76.7
12	9	10	47.4	52.6
Various	66	89		
TOTAL	175	250	41.2	58.8

MONTEREY PINE (Pinus radiata)

With the exception of about 20 cones examined at St. Mary's College, the rest were collected on Grizzly Peak (near Berkeley). Where necessary, the cones were soaked so that these statistics provide information regarding exceptions to the usually observed pattern.

TREE	8R	8L	NS	%8R	%8L	$\% \mathrm{NS}$
1	5	5		50.0	50.0	
2	13	8		61.9	38.1	
3	20	23	5	41.7	47.9	10.4
Various	148	132	2			
TOTAL	186	168	7	51.5	46.5	1.9

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BISHOP PINE

Cones were studied on trees north of Port Ross. The fact that there are no exceptions to the pattern is significant.

TREE	8R	8L	%8 m R	%8L
1	21	9	70.0	30.0
2	22	16	57.9	42.1
3	2	15	11.8	88.2
4	15	7	68.2	31.8
5	50	32	61.0	39.0
Various	3	1		
TOTAL	113	80	58.5	41.4

BEACH PINE (Pinus contorta)

The trees studied were found on Albion Ridge some ten miles south of Fort Bragg. The fact that there are no exceptions to the standard patterns is significant.

TREE	$8\mathbf{R}$	8L	$\%8\mathrm{R}$	%8L
1	28	21	57.1	42.9
2	26	27	49.1	50.9
3	13	23	36.1	63.9
4	23	23	50.0	50.0
Various	68	22		
TOTAL	158	116	57.7	42.3

DOUGLAS FIR (Pseudotsuga Menziesii according to Munz)

Cones were collected from trees near St. Helena in Napa County.

TREE	5R	5L	NS	$\%5\mathrm{R}$	$\%5\mathrm{L}$	%NS
1	54	45		54.5	45.5	
2	38	42	1	46.9	51.9	1.2
3	31	60		34.1	65.9	
Various	47	40				
TOTAL	170	187	1	47.5	52.2	0.3

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CONCLUSIONS AND OBSERVATIONS

On the basis of this very qualitative investigation it is possible to arrive at a few conclusions and observations.

1. Overall for most species of pines there appears to be a tendency to have about as many right as left spiral cones. (Right and left are purely relative to the definition of this investigation.) Overall for the pine cones (apart from Douglas fir) considered in this study, of the 8R-8L groups, 50.9% were 8R and 49.1%8L; of the 5R-5L groups, 51.1% were 5R and 48.9% were 5L. (The exceptional cases are not included in these percentages.)

2. For any tree with ten or more cones, there were always both right and left spiral cones.

3. If the probability of right and left spiral cones is approximately 50%, then there were certain trees which seemed to deviate from this figure to a marked degree. Apparently they had a tendency to produce more of one type than the other.

4. For lodgepole, Jeffrey, silver, yellow, one-needled pinyon, foxtail, knobcone, Monterey, bishop and beach pines, there were 74 exceptional cones out of 4290 which is 1.7%. If the foxtail pine is eliminated, the percent drops to 1.0. Note also that knobcone, bishop and beach pines showed no exceptions for the samples considered.

5. Occasionally one finds a tree with a high percent of cones which deviate from the usual pattern.

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