FIBONACCI SYSTEM IN AROIDS

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INTRODUCTION

The Aroids (family Araceae) are a group of attractive ornamental plants which include the very familiar Aglacnemas, Alocasias, Anthuriums, Arums, Caladiums, Colocasias, Dieffenbachias, Monsteras, Philodendrons, Scindapsuses and Spathyphyllums. The numerous species and varieties of some of them bearing leaves of different sizes, shapes and colors, and a most attractive fleshy spathe (cover) that surrounds the cylindrical inflorescence (flower bunch) spoken of as the spadix, are popular throughout the world. The spadix stands distinctive against the background of the hoodlike spathe. The fleshy cylindrical column of the spadix bears a multitude of stalkless flowers. Usually there are three kinds of flowers in a spadix — the females which occupy the lowest portion, the males the topmost region and the bisexual flowers located between the males and females. One would hardly believe that these flowers are packed in a mathematical pattern.

In most Aroids, clear spirals are discernible on the arrangement of the flowers. The numbers of these spirals generally synchronize with Fibonacci Numbers. But in some species they do not. Observations were made on a number of spadices each of 20 species of Aroids at the Royal Agri-Horticultural Society's garden at Calcutta in 1970, and the numbers of spirals in each of them recorded. In each inflorescence which follows the Fibonacci system positively and where the flowers are arranged in spirals, one can trace out the spirals running clockwise as well as counter-clockwise. The spirals in a spadix numerically always happen to be two consecutive Fibonacci numbers. According to the size of the inflorescence, the numbers of spirals generally vary, the thinner ones having smaller numbers. Moreover, in a species where the numbers of spirals are, say, 5 and 8, some individuals have the five spirals moving clockwise (and the eight spirals, counter-clockwise). In other individuals of the same species, the reverse is the situation, which is like the left- and right-handedness reported for the coconut and other palms (Davis, 1971).

PRESENTATION OF DATA

A number of spadices from six _Anthurium_ species were examined. In addition to recording the numbers of spirals veering to the left and to the right, the total length of the spadix and its maximum thickness were also measured. The spadix of _Anthurium_ is slender, elongated and uniform bearing only bisexual flowers (Fig. 1). Data on _Anthurium macrolobium_ are presented in Table 1.

Table 1
Data on 20 Spadices of Anthurium Macrolobium

No.	Spirals Left	Spirals Right	Length (cm)	Diameter (cm)
1	5	8	18.4	1.2
2	5	8	15.4	1.3
3	8	5	11.8	1.2
4	8	5	11.1	0.9
5	5	8	14.1	1.2
6	5	8	12.1	1.0
7	5	8	9.9	1.1
8	8	5	15.0	0.9
9	5	8	12.2	0.9
10	5	8	13.6	1.1
11	8	5	12.1	0.8
12	8	5	15.3	1.0
13	8	5	11.8	0.8
14	5	8	16.2	1.3
15	5	8	12.7	1.1
16	8	5	14.1	1.2
17	5	8	11.5	1.0
18	8	5	11.9	0.9
19	5	8	12.8	0.8
20	5	8	14.8	1.2
			266.8	20.9

L 5 and R 8 = 12
L 8 and R 5 = 8
Length of spadix = 13.34 cm
Maximum thickness = 1.05 cm

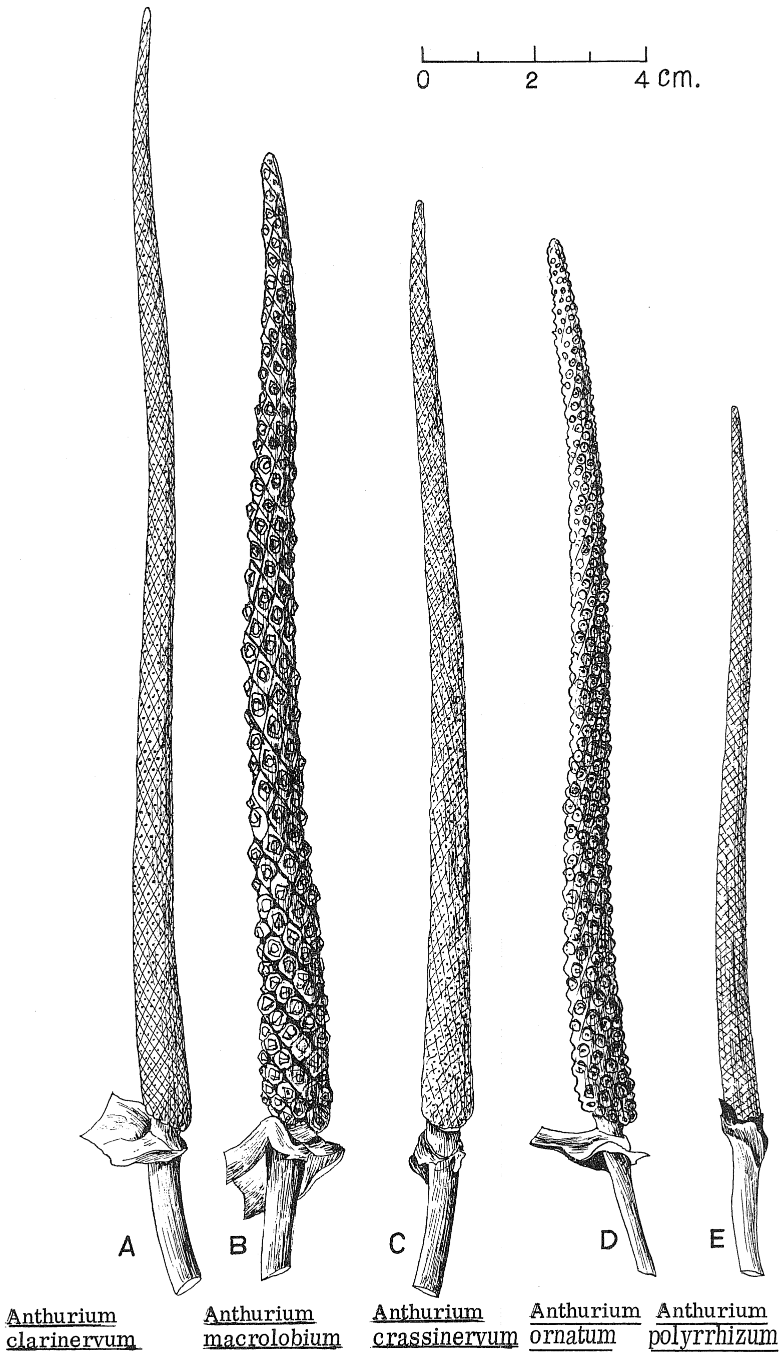

Fig. 1 Spadices of Five Species of Anthurium

Without an exception, all the spadices of Anthurium macrolobium bear floral spirals synchronizing the Fibonacci numbers 8 and 5, and they happen to be two consecutive stages in the sequence. The number of spadices having 5 left-veering spirals is higher than those with 8 left-veering spirals, but the difference is not statistically significant.

The summarized data on the spadices of Anthurium clarinervum are given below.

Data on Anthurium clarinervum

Left 8 and Right 13	=	5
Left 13 and Right 8	=	2
Left 8 and Right 5	=	1
Length of spadix	=	14.2 cm
Maximum thickness	=	0.6 cm

While a great majority of the spadices in the above species showed 13 and 8 spirals, one manifested the next lower numbers, i. e., 8 and 5.

In Anthurium ornatum, all the spadices examined showed 13 and 8 spirals as per data shown below.

Data on Anthurium ornatum

Left 8 and Right 13	=	8
Left 13 and Right 8	=	4
Others	=	Nil
Length of spadix	=	9.43 cm
Maximum thickness	=	0.89 cm

In Anthurium polyrrhizum and Anthurium andraeanum rubrum, the numbers of floral spirals are fixed at 13 and 8 and no exception was met with as the data in Table 2 show.

Table 2

Number of Spadices for Anthurium polyrrhizum and andraeanum rubrum

	Number of Spadices	
Spiral numbers	polyrrhizum	andraeanum rubrum
L 8 and R 13	6	6
L 13 and R 8	3	4
Others	Nil	Nil
Length of Spadix	7.43 cm	7.52 cm
Maximum Thickness	0.71 cm	0.73 cm

Anthurium crassinervum is a species having a larger spadix, and accordingly, it shows still higher numbers of floral spirals, i.e., 13 and 21 as per data shown below.

Data on Anthurium crassinervum

Left 13 and Right 21	=	8	
Left 21 and Right 13	=	6	
Left 11 and Right 18	=	1	(Lucas)
Length of spadix	=	15.04 cm	
Maximum thickness	=	1.39 cm	

The aberrant spadix bears spiral numbers short of 2 (left) and 3 (right) to match the rest. Incidentally, these are also Fibonacci numbers.

Schizocasia poteia, with an exception of one spadix, shows 13 and 8 floral spirals.

A Spathyphyllum spadix also conforms more or less to the Fibonacci system by displaying 8 and 5 floral spirals in a great majority of the spadices as per data given below.

Data on Spathyphyllum Spadices

Left 8 and Right 5	=	10
Left 5 and Right 8	=	3
Left 6 and Right 5	=	1
Left 6 and Right 7	=	1
Left 6 and Right 8	=	1
Length of spadix	=	7.66 cm
Maximum thickness	=	0.76 cm

Out of the total 16 spadices examined, three did not conform to the Fibonacci system, even though in two of them, the right-veering spirals numbering 5 and 8 show their affinity to the Fibonacci system.

The four species of Dieffenbachia whose spadices were examined (Dieffenbachia picta viridis, Dieffenbachia picta, Dieffenbachia dagneus, and an unidentified species) have their female flowers, which are much larger and sparser, are borne only on one side of the flattened spadix, the opposite side being fused with the spathe. However, it was possible to make out 5 and 3 spirals out of the arrangement of these female flowers. Six of the 9 spadices of Diffenbachia dagneus had five left-moving spirals and three right-moving ones. In the rest, a reverse order was noticed.

In the species of Dieffenbachia studied, only very few bisexual flowers were present, and this region of the column is considerably barren. The upper region consisting of the closely packed male flowers is quite prominent. Although regular spiral arrangement could be made out on these flowers, the numbers of spirals were not always Fibonacci numbers. Moreover, in many of them the numbers moving to the left and to the right were similar. The data on 18 spadices of Dieffenbachia picta viridis are shown below.

Data on Dieffenbachia picta viridis

Left 8 and Right 8	=	15
Left 8 and Right 7	=	2
Left 7 and Right 8	=	1
Length of spadix	=	12.39 cm

Similar data for Dieffenbachia picta given below relating to 17 spadices show a greater variation.

Data on Dieffenbachia picta

Left 7 and Right 7	=	1
Left 6 and Right 6	=	3
Left 3 and Right 4	=	1
Left 5 and Right 6	=	5
Left 6 and Right 5	=	6
Left 7 and Right 6	=	1

In some species like Aglaonema commutation and Arisaema ringens, a number of spadices each were examined. But it was very difficult to make out regular spirals in them.

In another unidentified species of Aglaonema, the following data were obtained from 13 spadices. Similar data on Syngonium spadices which closely resembles Aglaonema spadices are also shown in Table 3 with those for Aglaonema.

Table 3
Data for Aglaonema Spadices and Syngonium Spadices

Spiral numbers	Aglaonema Spadices	Syngonium Spadices
L 5 and R 5	9	5
L 5 and R 6	1	4
L 5 and R 7	Nil	1
L 5 and R 8	1	Nil
L 6 and R 5	1	2
L 6 and R 6	Nil	5
L 7 and R 5	1	Nil
L 7 and R 6	Nil	1
L 7 and R 7	Nil	2
Total	13	20
Length of spadix	2.96 cm	3.26 cm
Maximum thickness	0.7 cm	1.83 cm

The varying patterns in the number of floral spirals in an Alocasia spadix and in Alocasia indica mettalica are given in Table 4.

Table 4
Data on Two Alocasia Spadices

Alocasia Spadix	Alocasia indica mettalica
L 5 and R 5 = 1	L 9 and R 9 = 2
L 6 and R 6 = 4	L 11 and R 11 = 1
L 7 and R 7 = 4	L 9 and R 8 = 1
L 5 and R 6 = 1	L 9 and R 10 = 1
L 6 and R 7 = 2	L 10 and R 11 = 3
L 6 and R 8 = 1	L 11 and R 9 = 1
L 7 and R 6 = 3	L 12 and R 11 = 1
L 7 and R 8 = 1	
L 8 and R 7 = 1	
Total 18	Total 10
Length of spadix = 3.08	Length of spadix = 17.65 cm
Maximum thickness = 1.76 cm	Maximum thickness = 0.98 cm

A species of Caladium also showed irregularities by exhibiting 9 to 12 spirals, none of them synchronizing a Fibonacci number.

Given below are data on Philodendron spirals relating to 15 spadices.

Data on Philodendron spirals

L 12 and R 12	=	2
L 13 and R 13	=	7
L 15 and R 15	=	1
L 12 and R 13	=	1
L 14 and R 13	=	2
L 14 and R 15	=	1
L 15 and R 14	=	1
Length of spadix	=	12.88 cm
Maximum thickness	=	2.89 cm

In this species, a majority of the spadices possess equal numbers of floral spirals running clockwise and counter-clockwise. Also in many spadices, the number of spirals do not synchronize Fibonacci numbers.

DISCUSSION

To the list of pine cones (Brousseau, 1968), palms (Davis, 1971), sunflowers and the very many situations in plants arising out of alternate arrangement of leaves, may be included the Aroids so far as their affinity to the Fibonacci sequence is concerned.

Among the aroids, the several species of Anthurium show, without an exception, floral spirals whose numbers synchronize the Fibonacci numbers. This is due to the fact that any two consecutive flowers subtend between them an angular deflection which make with the remaining angle to complete one full revolution, the familiar golden ratio. At the tip of these spadices, the flowers end in smaller numbers of spirals. From the very last flower which can be easily made out in these cases, its nearness to the just preceding one can be made out. All the five spadices in Fig. 1 show this arrangement. Moreover, these compact flower-bunches although taper smoothly, there is no irregularity of any sort in any region. On the other hand, the five spadices shown in Fig. 2 are uneven, the last one (Monstera deliciosa) being an exception. By careful observation of the female flowers, spiral numbers which

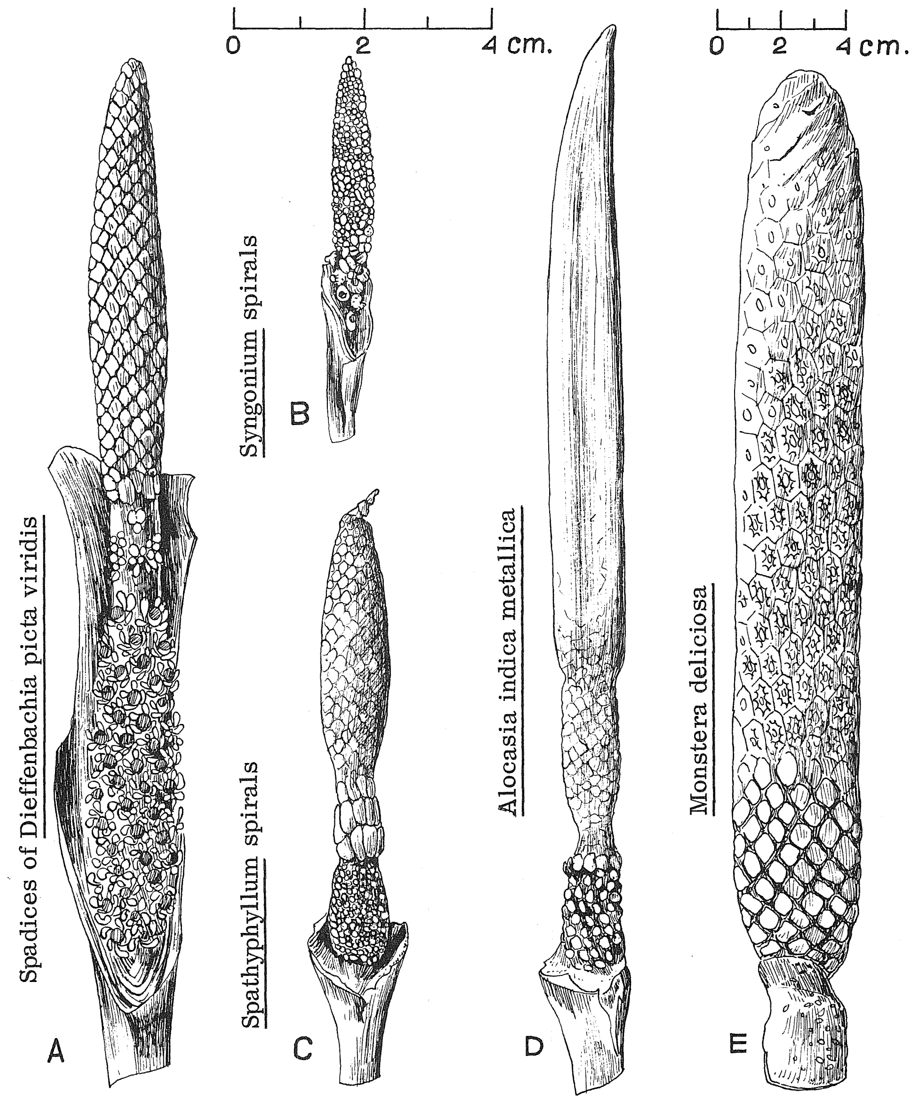

Fig. 2 Spadices of Dieffenbachia picta viridis

synchronize Fibonacci numbers can be made out. The bisexual flowers distributed in the narrow region are devoid of any obvious spirals. Moreover, at this region, the spadix remains considerably narrow. It is at this region, presumably, the angle between consecutive flowers undergo a change. As a result, the male flowers at the upper region either do not fall in regular spirals, or the spirals do not conform to Fibonacci numbers. In the same species, different spadices show much differing numbers of spirals. A better understanding of the phyllotaxis in these species may be essential to study the cause of such a variation.

Pineapples as well as the male and female reproductive bodies (cones) of many species of Cycas show clear spirals in the arrangement of the individual fruits and generative leaves respectively, and these numbers are always Fibonacci numbers. In small pineapples (Fig. 3), 3 and 5 spirals are visible. As in Aroids and palms, the 3 spirals in a pineapple may veer clockwise or counter-clockwise. In a larger variety, there are 5 and 8 spirals, and in still larger pineapples, there are 8 and 13 spirals. In some exceptionally larger ones, even 13 and 21 spirals can be made out.

Among Cycas, too, there are species which show 3 and 5 spirals and the numbers in some other species may go up to 13 and 21 as in the male cone of Cycas circinalis seen in Fig. 4.

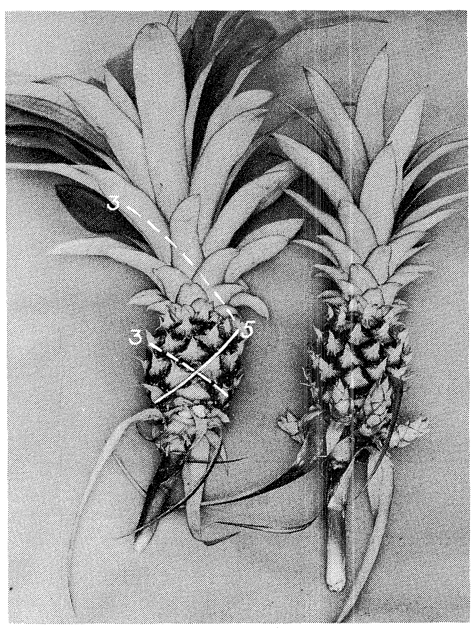

Fig. 3 Pineapples showing 3 and 5 spirals

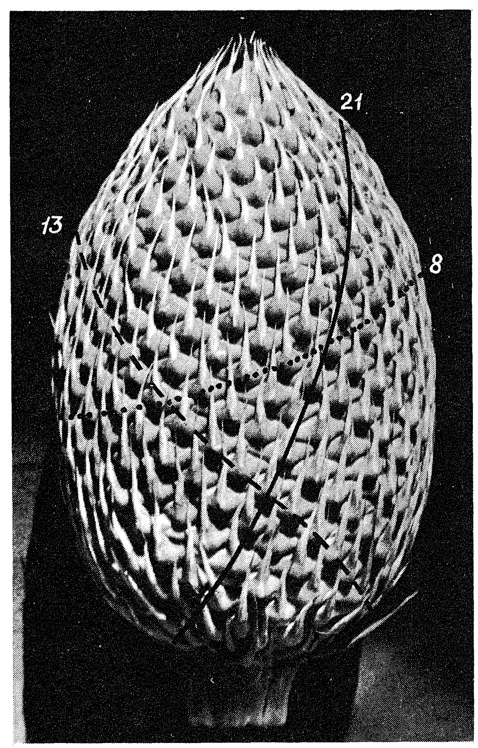

Fig. 4 Cones of Cycas circinalis showing 3 and 5 spirals

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